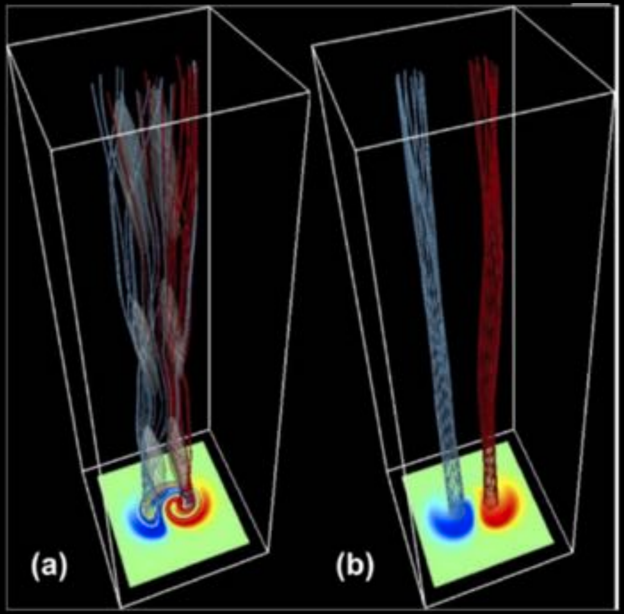
Capture the topology of resistive magnetic relaxation

NASA/GSFC/Solar Dynamics Observatory

On the solar surface, we can observe certain structures, e.g., coronal loops, evolving in time as they are illuminated by charged particles spinning along the magnetic field lines.

Numerical experiments of braided magnetic field show the complexity of the magnetic field reduces as it relaxes due to a small amount of resistivity (Yeates *et al.* 2010, 2015).

Here is one example. The entangled magnetic fields are shown throughout the volume with the projection of field line helicity (FLH), i.e., a topological measure, shown at the bottom plane:(a) initial state, (b) final state.



Russell *et al.* 2015

We take into account the major physical effects in the 3D simulations, and then construct an effective 2D model.

Density variation

$$\frac{D\rho}{Dt} = -\,\rho\nabla\cdot\boldsymbol{u},$$

Lorentz force

Fluid viscosity

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{\rho}\left[(\nabla\times\boldsymbol{B})\times\boldsymbol{B} - \nabla P - \nabla\cdot\boldsymbol{\sigma}\right],$$

Pressure term

$$\frac{D\boldsymbol{B}}{Dt} = (\boldsymbol{B}\cdot\nabla)\boldsymbol{u} - \boldsymbol{B}(\nabla\cdot\boldsymbol{u}) - \nabla\times(\eta\nabla\times\boldsymbol{B}),$$

Magnetic induction

Magnetic diffusion

$\cdots$ + other physical effects

The function f(x,y,t) now contains all the topological information.

FLH (topological measure)

$$f = \int \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}$$

Effective magnetic field

$$\widetilde{\boldsymbol{B}} = \nabla\times f(x,y,t)\hat{\boldsymbol{z}}$$

T. Arber *et. al.* (2001)

Fluid viscosity

Pressure term

$$\mu\nabla^2\boldsymbol{w} + (\nabla\times\widetilde{\boldsymbol{B}})\times\widetilde{\boldsymbol{B}} - \nabla P = 0,$$

Advection

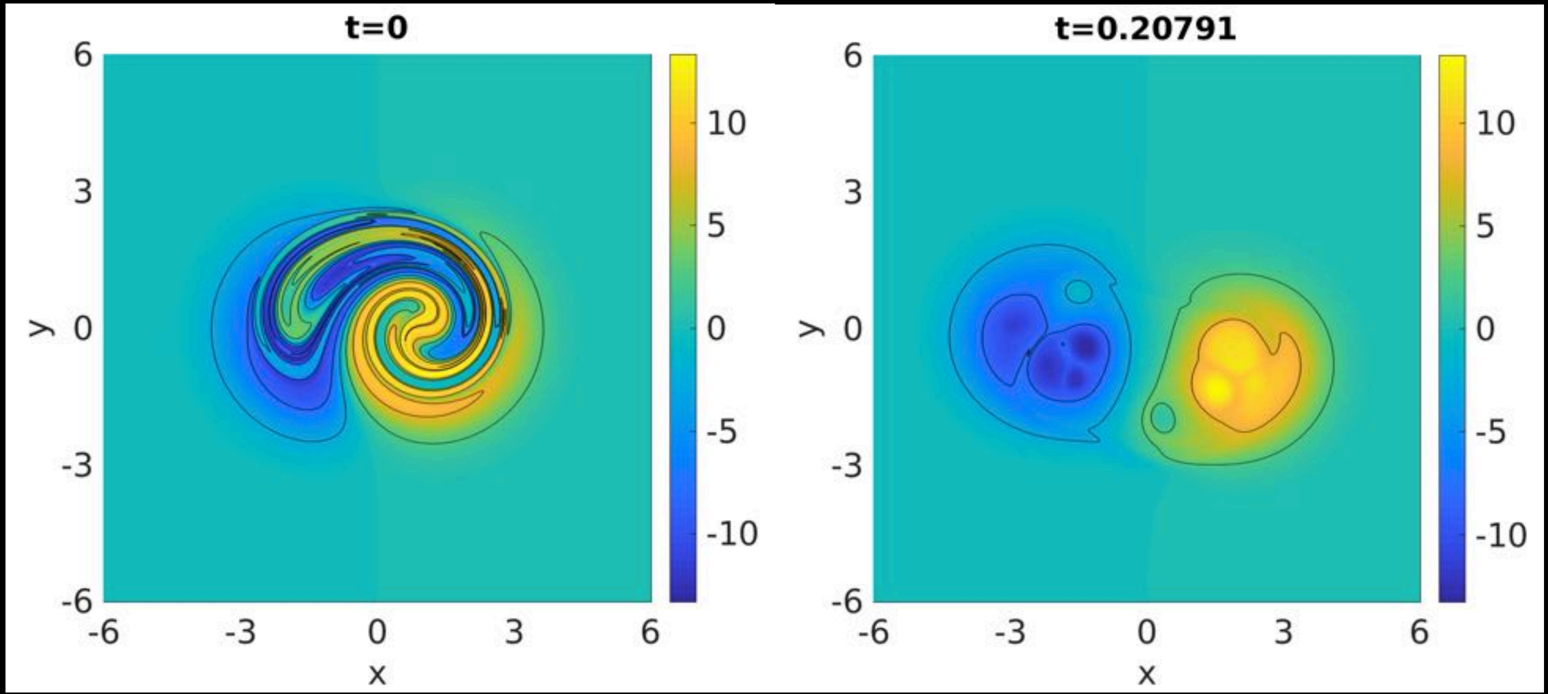
Effective Lorentz force

$$\frac{\partial f}{\partial t} + \boldsymbol{w}\cdot\nabla f = 0,$$

Incompressibility

$$\nabla\cdot\boldsymbol{w} = 0$$

It not only makes the computation more efficient, we can also test which physical effects are dominant in terms of changes in the topology in this reduced model.



We see this 2D model is able to capture the main topological feature of the original 3D simulation. The initially highly braided pattern (left) is able to relax to a simple state (right), which corresponds to the two oppositely twisted flux tubes.

We are in the process of testing more initial conditions, also to verify if another equivalent flow could lead to a similarly relaxed state. Ultimately, we hope the results of this effective 2D model can be applied not only to study resistive magnetic relaxation but also fluid mechanics in general, for example, the optimal transport theory.

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